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CONJUGATE HEAT TRANSFER IN MOVING FILAMENT BUNDLES WITH ARBITRARY BIOT NUMBERS

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Ananalysis of heat transfer in a bundle of filaments with arbitrary Biot numbers is carried out based on an approximate method proposed in the article.

When considering heat transfer in filament bundles being formed, the representation of a filament as a thermally thin body with a Biot number substantially smaller than unity is one of the basic assumptions [1, 2]. However, in a number of cases filament diameters can be rather large, and, therefore, the filaments cannot be considered thermally thin, and the temperature distribution along the filament cross-section is already inhomogeneous. In practice, in these cases, especially when phase transitions take place within the filament, a liquid core surrounded by a solidified shell is formed. The problem of inhomogeneity of the parameter distribution over the filament cross-section is even more closely related to diffusion processes. It is known that the diffusion coefficient in polymeric liquids is about $\sim 10^{-5}-10^{-7}$ cm^2/sec. In these cases the diffusional Peclet number is several orders of magnitude greater than the thermal one, which implies the inhomogeneity of the concentration profile in the zone of filament formation.

Determination of temperature or concentration profiles in a single filament presents no particular problems, since the problem allows a rigorous mathematical formulation and can be solved either analytically or numerically. When considering heat transfer processes in bundles, it is of critical importance to avoid numerical solutions of two-dimensional problems within each of filaments, since this leads to huge computational expenses. Therefore, the development of analytical and approximate methods making possible to simplify the formulation of the problem and reduce it to systems of one-dimensional equations is of topical interest. The objective of the present work was to construct approximate solutions for the thermal (diffusional) problem describing heat (mass) transfer within a filament and to incorporate the problem into the general heat (mass) transfer problem of a moving filament bundle.

1. Construction of Approximate Solution. Let us construct a solution using a semi-infinite band of thickness $2h$ as an example. The heat transfer equation can be written as

$$\frac{\partial T_f}{\partial \zeta} - \frac{\partial^2 T_f}{\partial n^2} = 0 , \tag{1}$$

$\zeta = \xi/\mathrm{Pe}_f$, $\xi = x/h$, and $n = y/h$. Boundary conditions are expressed as follows:

$$T_f(0, n) = T_{f0}(n) , \left. \frac{\partial T_f}{\partial n} \right|_{n=1} = -\mathrm{Bi}\,(T_{f_s} - T_m) . \tag{2}$$

It is known that at constant Bi and T_m, problem (1)-(2) has a simple analytical solution

$$T_f = \sum_{k=1} A_k \exp(-\sigma_k^2 \zeta) \cos(\sigma_k n) + T_m , \tag{3}$$

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where A_k are coefficients and σ_k are eigenvalues of the problem, which are determined from the relationship:

$$\sin \sigma_k = \frac{\mathrm{Bi}}{\sigma_k} \cos \sigma_k . \tag{4}$$

In a moving filament bundle, the temperature of the filtering flow, which varies both along and across the forming zone as the bunch moves, plays the role of temperature T_m. The Biot number can be considered constant on the main portion of motion of filaments in a tube, since hydrodynamic stabilization of the flow is established rather rapidly with low stretching. Therefore, we now consider problem (1)-(2) with the variable T_m.

We will use the multiparameter method [3], which is used in boundary-layer theory and makes it possible to obtain a solution at arbitrary boundary values, as the basis for construction of an approximate solution. In addition, solution (3)-(4) holds at arbitrary Bi values; therefore, it is worthwhile to use the form of this solution. According to the multiparameter method, the solution of equation (1) can be represented as the dependence $T_f(\zeta, n) = T_f(\zeta, n; f_1, f_2, ...)$, where f_i are form parameters related to the boundary conditions at $n = 1$. In the case under consideration, we introduce the function $t = T_f - f$, which satisfies the equation

$$\frac{\partial t}{\partial \zeta} - \frac{\partial^2 t}{\partial n^2} + f_1 + \sum_{i=1} \frac{\partial t}{\partial f_i} f_{i+1} = 0 . \tag{5}$$

Here f is a function of ζ, and form parameters $f_i = \partial^i f / \partial \zeta^i$. Due to the linearity of the equation with respect to t, the solution can be represented in the form $t = t_0 + t_1 + t_2$, where

$$\frac{\partial t_0}{\partial \zeta} - \frac{\partial^2 t_0}{\partial n^2} = 0 , \quad \frac{\partial t_1}{\partial \zeta} - \frac{\partial^2 t_1}{\partial n^2} = - f_1 , \quad \frac{\partial t_2}{\partial \zeta} - \frac{\partial^2 t_2}{\partial n^2} = - \frac{\partial t_1}{\partial f_1} f_2 . \tag{6}$$

Restricting ourselves to the first two equations of system (6), we write solutions in the form

$$t_0 = \sum_{k=1} B_{0k} \exp(-\sigma_k^2 \zeta) \cos(\sigma_k n) ,$$
$$t_1 = \frac{1}{2} f_1 n^2 + \sum_{k=1} B_{1k} \exp(-\sigma_k^2 \zeta) \cos(\sigma_k n) , \tag{7}$$

where σ_k also satisfy condition (4). In order for the solutions to be valid, one must satisfy the relationship following from boundary condition (2) at $n = 1$

$$\left(1 + \frac{1}{2} \mathrm{Bi}\right) \frac{df}{d\zeta} + \mathrm{Bi}\, f = \mathrm{Bi}\, T_m . \tag{8}$$

This expression plays the role of an equation for determination of the function f and the form parameter f_1. At a constant T_m, the equations $df/d\zeta = 0$ and $f = T_m$ hold, and the solution for t_0 transforms into (3). When T_m varies, the equation $f(0) = T_{m0}$ plays the role of a boundary condition when solving Eq. (8).

In order to estimate the applicability of the method proposed, we consider the following model problem. Let $T_m = a_0 \exp(\kappa\zeta)$ with a_0 and κ being constants. Then the exact solution of problem (1) with boundary conditions (2) ($T_{m0}(n) = 1$) is as follows ($\kappa > 0$):

$$T_f = B\, [\exp(\kappa n) + \exp(-\kappa n)] \exp(\kappa\zeta) + \sum_{k=1} A_k \exp(-\sigma_k^2 \zeta) \cos(\sigma_k n) ,$$

where

$$B = a_0 \left\{ [\exp(\sqrt{\kappa}) + \exp(-\sqrt{\kappa})]\, \mathrm{Bi} + \sqrt{\kappa}\, [\exp(\sqrt{\kappa}) - \exp(-\sqrt{\kappa})] \right\}^{-1} \mathrm{Bi} ,$$

TABLE 1. Temperature Values Obtained from Analytical and Approximate Solutions

Bi	ζ	$T_{\text{mean an}}$	$T_{\text{a an}}$	$T_{\text{s an}}$	$T_{\text{mean ap}}$	$T_{\text{a ap}}$	$T_{\text{s ap}}$
1	0.4	1.07	1.03	1.16	1.11	1.06	1.20
	0.8	1.27	1.17	1.50	1.35	1.24	1.57
	1.2	1.67	1.49	2.06	1.79	1.59	2.17
	1.6	2.32	2.02	2.94	2.50	2.19	3.11
	2.0	3.32	2.87	4.29	3.59	3.11	4.54
10	0.4	1.18	1.08	1.41	1.23	1.13	1.43
	0.8	1.63	1.42	2.08	1.69	1.49	2.10
	1.2	2.37	2.04	3.09	2.46	2.13	3.12
	1.6	3.52	3.00	4.61	3.63	3.12	4.65
	2.0	5.23	4.46	6.87	5.40	4.63	6.93
100	0.4	1.22	1.11	1.48	1.26	1.14	1.49
	0.8	1.72	1.49	2.21	1.77	1.54	2.21
	1.2	2.52	2.16	3.30	2.59	2.23	3.30
	1.6	3.75	3.19	4.92	3.84	3.30	4.92
	2.0	5.59	4.76	7.33	5.71	4.90	7.34
1000	0.4	1.27	1.17	1.49	1.26	1.14	1.49
	0.8	1.74	1.52	2.22	1.78	1.55	2.22
	1.2	2.55	2.18	3.32	2.60	2.24	3.32
	1.6	3.78	3.22	4.95	3.86	3.31	4.95
	2.0	5.62	4.79	7.38	5.75	4.92	7.38

$$A_k = 2\left(1 + \frac{\sin 2\sigma_k}{2\sigma_k}\right)^{-1}\left\{\frac{\sin \sigma_k}{\sigma_k} - \right.$$

$$\left. - \frac{[\exp(\sqrt{\kappa}) + \exp(-\sqrt{\kappa})]\,\mathrm{Bi} + \sqrt{\kappa}\,[\exp(\sqrt{\kappa}) - \exp(-\sqrt{\kappa})]}{\sigma_k^2 + \kappa} B \cos \sigma_k \right\}.$$

In the approximate solution we have

$$B_{0k} = 2\,(1 - a_0)\left(1 + \frac{\sin 2\sigma_k}{2\sigma_k}\right)^{-1}\frac{\sin \sigma_k}{\sigma_k},\quad B_{1k} = 0,$$

and in this case

$$f = a_0\left(1 - \frac{2\,\mathrm{Bi}}{2\,\mathrm{Bi} + \kappa\,(2 + \mathrm{Bi})}\right)\exp\left(\frac{2\,\mathrm{Bi}\,\kappa}{2 + \mathrm{Bi}}\right) + a_0\frac{2\,\mathrm{Bi}\exp(\kappa\zeta)}{2\,\mathrm{Bi} + \kappa\,(2 + \mathrm{Bi})}.$$

Table 1 presents values of mean temperatures and axial and surface temperatures of plates obtained from analytical and approximate solutions ($T_{\text{mean an}}$, $T_{\text{a an}}$, and $T_{\text{s an}}$ correspond to the analytical solution, and $T_{\text{mean ap}}$, $T_{\text{a ap}}$, and $T_{\text{s ap}}$ correspond to the approximate one). It follows from the table that the values of the quantities presented are correspondingly close to each other. Thus, results of calculations show that, based on the approximations introduced, the method makes it possible to calculate the internal temperature distribution within a filament when the two-dimensional problem (1) is reduced to the one-dimensional one (8).

2. Heat Transfer in a Bundle of Moving Filaments. Let us consider the problem on heat transfer in a bundle of moving filaments. To describe the internal heat transfer in a filament, we will use the equation in the following form:

$$\mathrm{Pe_f}\frac{\partial T_f}{\partial \xi}=\frac{\partial^2 T_f}{\partial n^2}+\frac{\partial T_f}{n\partial n}, \tag{9}$$

in this case ($n = r/r_f$)

$$T_f(0, n) = T_{f0}, \qquad \left.\frac{\partial T_f}{\partial n}\right|_{n=1} = \mathrm{Bi}\,(T_{f_s} - T_{fl}). \tag{10}$$

According to the above, we present the solution of the internal problem (9)-(10) with regard for the two approximations as follows:

$$T_f = f + \frac{\mathrm{Pe_f}}{4}\frac{df}{d\xi}n^2 + \sum_{k=1} B_k \exp(-\sigma_k^2\xi/\mathrm{Pe_f})\, J_0(\sigma_k n), \tag{11}$$

where $\mathrm{Bi} = 2\mathrm{Bi}T_{f0}/((\mathrm{Bi}^2 + \sigma_k^2)\sigma_k J_1(\sigma_k))$; in this case the function f satisfies the equation

$$\frac{\mathrm{Pe_f}}{2}\left(1+\frac{1}{2}\mathrm{Bi}\right)\frac{df}{d\xi} + \mathrm{Bi}\,f = \mathrm{Bi}\,T_{fl} \tag{12}$$

with boundary conditions $f(0) = T_{fl\,0}$, and σ_k is determined from the relationship

$$J_1(\sigma_k) = \frac{\mathrm{Bi}}{\sigma_k}J_0(\sigma_k).$$

In order to perform calculations of heat transfer in filament bundles, we will use the mathematical model of filtration flow which we developed in [4, 5]. We consider the bundle as either being rather thick or consisting of a number of filamen s, which results in screening of immediate boundary conditions. In this case, as has been shown in [5], when bundles move in a tube, hydrodynamic stabilization is established relatively rapidly, and a regime is established in a bundle such that the Nusselt numbers for each of filaments in the bundle can be considered as virtually equal up to the very boundary of the bundle. This holds in full measure for heat transfer. Therefore, results obtained from the solution of the problem considered can also be used for estimation of mass transfer parameters. Thus, for a hydrodynamically stabilized regime, the heat transfer equation can be simplified and written in the form

$$n_*^{-1}\,\mathrm{Pe}\frac{\partial T_{fl}}{\partial \xi} = 2\,\mathrm{Nu}\,(T_{f_s} - T_{fl}). \tag{13}$$

Using expressions (11) and (12) and Eq. (13), one can obtain the following analytical solution of the conjugate heat transfer problem:

$$\tau_f = \frac{T_f - T_{fl0}}{T_{f0} - T_{fl0}} = \frac{\mathrm{Bi}}{2+\mathrm{Bi}}\left(\sum_{k=1} A_k\exp(-\sigma_k^2\xi/\mathrm{Pe_f}) + A\exp(-\chi\xi/\mathrm{Pe_f})\right) + f,$$

$$\tau_{fl} = \frac{T_{fl} - T_{fl0}}{T_{f0} - T_{fl0}} = \sum_{k=1} A_k\exp(-\sigma_k^2\xi/\mathrm{Pe_f}) + A\exp(-\chi\xi/\mathrm{Pe_f}) + f,$$

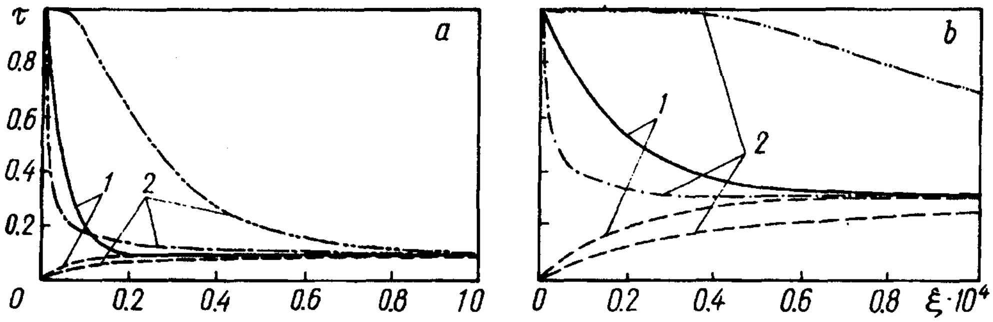

Fig. 1. Changes in temperatures of the medium and filaments in an infinitely thick bundle (Pe = 100): a: 1) Bi = 0.1, $\mathrm{Pe_f} = 10^2$, 2) Bi = 10, $\mathrm{Pe_f} = 10^4$; b: 1) Bi = 0.1, $\mathrm{Pe_f} = 5 \cdot 10^2$; 2) Bi = 10, $\mathrm{Pe_f} = 5 \cdot 10^4$.

$$\tau_{\mathrm{a}} = \frac{T_{\mathrm{a}} - T_{\mathrm{fl0}}}{T_{\mathrm{f0}} - T_{\mathrm{fl0}}} = \sum_{k=1} A_k \exp(-\sigma_k^2 \xi/\mathrm{Pe_f}) + f, \tag{14}$$

$$f = -\frac{4\,\mathrm{Bi}}{2 + \mathrm{Bi}} \left(\sum_{k=1} \sigma_k^{-2} A_k \exp(-\sigma_k^2 \xi/\mathrm{Pe_f}) + \kappa^{-1} A \exp(-\chi\xi/\mathrm{Pe_f}) \right),$$

where

$$\chi = \frac{4}{2 + \mathrm{Bi}} \left(\mathrm{Bi} + n_* \mathrm{Nu} \frac{\mathrm{Pe_f}}{\mathrm{Pe}} \right), \quad A = -\sum_{k=1} A_k,$$

$$A_k = \frac{2n_* \mathrm{Pe}\,\mathrm{Nu}}{\mathrm{Pe} \left\{ \frac{4}{2 + \mathrm{Bi}} \left(\mathrm{Bi} + n_* \mathrm{Nu} \frac{\mathrm{Pe_f}}{\mathrm{Pe}} \right) - \sigma_k^2 \right\}} B_k.$$

Figure 1 shows changes in the temperature of filaments and the medium along the axis of motion of the bundle (solid curves correspond to temperatures obtained from the one-dimensional theory for a thermally thin filament). In addition, Fig. 1a presents results of calculations at Nu = 0.35 and Bi = 0.1 and 10. In this case, the Peclet number $\mathrm{Pe_f}$ for the moving filament was taken to be equal to 10^2 and 10^4 respectively, and the Peclet number Pe in the interfilament space was taken to be equal to 10^2. This relationship between the Biot and Peclet numbers ensures constancy of the total heat content of the medium and filaments. In the former case, (Bi = 0.1), the temperature curves corresponding to the surface ($\tau_{\mathrm{f_s}}$) and axial ($\tau_{\mathrm{f_a}}$) temperatures of the filaments coincide in the scale of the figure with the curve obtained from the one-dimensional theory (τ_{1D}). The same coincidence is observed for the curves for the temperature of the medium (the filtration temperature of the medium is shown by dashed curves). For Bi = 10, the curves of the surface temperature of the filament (dash-dot curve) and on its axis (dash-double dot curve) diverge at the initial instant; in this case, on a certain portion, $\tau_{\mathrm{f_a}}$ has a constant value equal to the initial temperature of the filament. The curve of $\tau_{\mathrm{f_s}}$ first drops somewhat more steeply than τ_{1D}, since the heat from central areas of the filament does not have time to reach the surface. However, when a certain temperature gradient is developed, a regime is established within the filament such that $\tau_{\mathrm{f_a}}$ and $\tau_{\mathrm{f_s}}$ slowly approach each other. The important feature of the presented calculation example is the fact that all the curves converge to one and the same value, which is determined from the heat balance condition. This substantiates to some extent the efficiency of the method.

In the following variant illustrated by Fig. 1b, we also assumed that Nu = 0.35 and Bi = 0.1 and 10, and Pe = 10^2. The values of $\mathrm{Pe_f}$ were taken to be equal $5 \cdot 10^2$ and $5 \cdot 10^4$, respectively, which also conserves the total heat content with a varying Biot number.

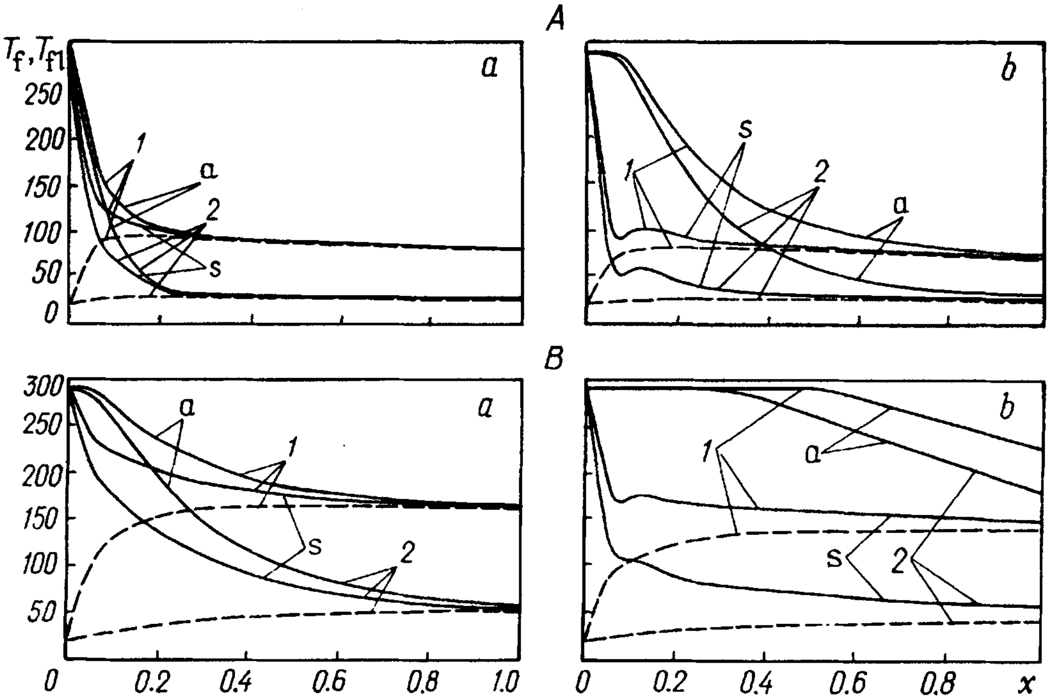

Fig. 2. Changes in temperatures of the medium and filaments in a bundle: A: a) Bi = 1, $Pe_f = 10^3$; b) Bi = 10, $Pe_f = 10^4$; B: a) Bi = 1, $Pe_f = 5\cdot10^3$; b) Bi = 10, $Pe_f = 5\cdot10^4$. T_f, T_{fl}, °C; x, m

The example presented also reveals the effect of the Biot number on the heat transfer process with conservation of total heat content. The character of variation of the curves is the same as in Fig. 1a. At Bi = 0.1, all the curves coincide with the corresponding curves obtained from the one-dimensional theory. At Bi = 10, substantial divergence takes place for τ_{f_s} and τ_{f_a}. In this case, if τ_{f_s} is close to the equilibrium temperature by the end of the heat transfer zone presented, the temperature on the filament axis is comparably far from it. Nevertheless, as in the first example, here all the curves converge on their equilibrium temperature. Thus, the Biot number affects the character of behavior of the curves: the curves converge at small Bi and diverge at large Bi. The relationship between Pe and Pe_f affects the length of establishment of equilibrium and the value of the equilibrium temperature. In particular, the equilibrium temperature and its establishment length increase with Pe_f.

As is evident from the results, the method proposed can be used for determination of the temperature distribution within a filament when it moves in a bundle. However, it should be pointed on that the given method can be successfully applied if the neglected terms in the form of form parameters of higher orders are smaller than the retained ones. When estimating the accuracy of the method for the number of approximations under consideration, one should take into account the value of the term $Pe_f f_2 \partial t/\partial f_1$ in Eq. (5), which in the case under consideration has the form $\varphi = Pe_f(\partial f_1/\partial\xi)/48$ in the expression for the temperature T_{f_s}.

In the examples presented, as is shown by calculations, this value is insignificant compared to T_{f_s}. When the values of φ and T_{f_s} are comparable, an error is possible, which is natural for this method and imposes restrictions on its applications. An error can be induced by a large longitudinal temperature gradient. However, it should be noted that heat and mass transfer in filament bundles being formed takes place in a rather smooth manner; therefore, this approximate method can be used to solve a number of problems of practical interest.

We also carried out calculations of the temperatures of the filaments and gas in bundles moving in a tube. The governing parameters had the following values: the tube radius $R_t = 0.1$ m, the bundle radius $R_b = 0.05$ m, the filament radius $r_f = 0.125\cdot10^{-3}$ m, the filament velocity $U_f = 0.3$ m/sec, the number of filaments $N = 100$, the initial filament temperature $T_f = 290$°C, the initial gas temperature $T_{fl} = 20$°C, and the wall temperature $T_w = 20$°C. We considered a model problem with air as the medium, and the material was assumed to have parameters providing the following values of the Biot and Peclet numbers: Bi = 1, $Pe_f = 10^3$; Bi = 10, $Pe_f = 10^4$; and Bi = 10, $Pe_f = 5\cdot10^4$.

Figure 2 shows curves of temperature variations on the surface (s) and in the center (c) of a filament positioned both at the boundary (curves 2) and on the axis (curves 1) of the bundle along the forming zone. The solid curves represent the filament temperatures, and the dashed curves correspond to the temperatures of the medium. The set of Biot and Peclet numbers in Fig. 2A provides, other parameters being constant, the same initial heat content of the filaments in the bundle. The same, but with another initial heat content, is also provided in the variant shown in Fig. 2B.

It is evident from Fig. 2 that the lateral heat removal by the apparatus wall and the dynamics of gas motion in the bundle affect substantially the central portion of the bundle. As opposed to an infinitely thick bundle, an equilibrium temperature is virtually not observed in this problem. Here we can speak about quasi-equilibrium quantities, which for a constant initial heat content, differ from each other depending on the Bi number. In this case, with a decreasing Biot number (an increasing thermal conductivity coefficient of the material) the quasi-equilibrium temperature at the center of the bundle increases at the same values of the longitudinal coordinate, which is due to more intense heat exchange. An increase in the initial heat content of the filaments (see Fig. 2B), as is evident also from Fig. 1, results in an increase in the temperatures of the gas and the filaments in the bundle.

The examples presented correspond to an inhomogeneous initial velocity of the gas in the apparatus, which is characteristic for actual devices (the gas velocity in the bundle in the initial cross-section is lower than the mean velocity in the apparatus). If one takes constancy of the velocity over the cross-section as an initial distribution, and by this means reduces the effect of dynamics on heat transfer (removes gas flows within the bundle), then, as has been shown by calculations (not presented here), for the Biot numbers taken in the paper, the quasi-equilibrium temperatures on the bundle axis become virtually equal. This characterizes the effect of dynamics on heat transfer and points to rather strong thermal blocking of the internal filaments.

NOTATION

x, y, Cartesian coordinate system; x, r, cylindrical coordinate system; r_f, filament radius; T_f, filament temperature; T_m, temperature of the medium; T_{fl}, temperature of the filtration flow; T_{f_s}, T_{f_a}, temperature on the surface and axis of a filament; T_{f0}, T_{fl0}, initial temperature values; $n = r_f/r_\Delta$, r_Δ, external radius of a cell in a filament bundle; Nu, the Nusselt number; $\mathrm{Pe} = \rho c_p r_\Delta U_{fl}/\lambda$, the Peclet number of the filtration flow; ρ, medium density; c_p, specific heat of the medium; λ, thermal conductivity of the medium; U_{fl}, filtration velocity; $\mathrm{Pe}_f = \rho_f c_{pf} r_f U_f/\lambda_f$, the Peclet number for the filament; ρ_f, material density; c_{pf}, specific heat of the material; λ_f, thermal conductivity in the filament; U_f, filament velocity.

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